**Homework 2 Solutions**

Zezhou Li

10405265

**Problem 1:**

1. Calculate the maximum likelihood estimation of parameter *θ*:

From the concept we know that the log-likelihood function is defined as the following:

and

thus

To derive the that maximum the , we should let = 0. Thus,

Finally,

So the maximum likelihood estimation of parameter *θ* is

2. Calculate the Bayesian estimation of parameter *θ* :

First, compute

Here, and .

Thus,

Here is bigger than 0 and fixed which is independent of , thus put it into .

To calculate , again using the derivative of ,

Again, let ,

Two results are the same.

**Problem 2:**

1. First, we should combine two training sample set {D1,D2}, then setup the scatter matrix as the following:

Using Matlab to calculate:

|  |
| --- |
| clear;  clc;  D1=[[1;2],[-3;-1],[4;5],[-1;1]];  D2=[[0;-2],[5;2],[-1;-4],[3;1]];  D = [D1,D2];  m=[sum(D(1,:));sum(D(2,:))]/8;  S=[0 0;0 0];  for k=1:8  T = D(:,k)-m;  S = S + T\*T';  end |

Finally, .

After getting S, we set where , we should find out the largest eigenvalue of S and determine e from the largest eigenvalue.

Using Matlab to find the eigenvalues of S:

|  |
| --- |
| eig(S) |

We get the eigenvalues of S are: 17 and 91. We choose the largest one which is 91 to calculate e.

We know that the following equation must be true:

We have , thus we can get e:

We also know that , thus:

Same as:

Then to calculate , we need to know :

Using Matlab:

|  |
| --- |
| e=[sqrt(2)/2;sqrt(2)/2];  a = [];  for i=1:8  a(i)=e'\*(D(:,i)-m);  end |

Then again using Matlab to calculate :

|  |
| --- |
| xk=[];  for i=1:8  xk=[xk,m+a(i)\*e];  end |

2. To calculate the desired w, first calculate Si.

Using Matlab:

|  |
| --- |
| S1=[0 0;0 0];  S2=[0 0;0 0];  for i=1:4  T1=D1(:,i)-m1;  T2=D2(:,i)-m2;  S1 = S1+T1\*T1';  S2 = S2+T2\*T2';  end |

We get , .

Then

.

We know that

Here

Thus using Matlab:

|  |
| --- |
| m1=[sum(D1(1,:));sum(D1(2,:))]/4;  m2=[sum(D2(1,:));sum(D2(2,:))]/4; |

We get:

Then we can get w:

|  |
| --- |
| w=Sw^(-1)\*(m1-m2) |

Since we have w, we can then calculate .

Using Matlab:

|  |
| --- |
| w'\*D |